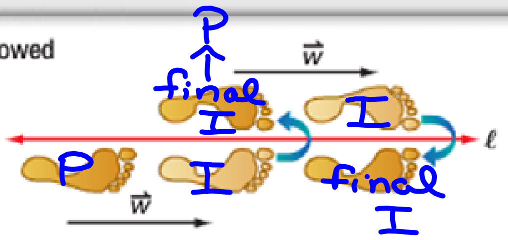
1 Glide Reflections When a transformation is applied to a figure and then another transformation is applied to its image, the result is called a **composition of transformations**. A glide reflection is one type of composition of transformations.

KeyConcept Glide Reflection

A **glide reflection** is the composition of a translation followed by a reflection in a line parallel to the translation vector.

Example

The glide reflection shown is the composition of a translation along $\vec{w}$ followed by a reflection in line ℓ.



Example 1 Graph a Glide Reflection

Triangle JKL has vertices $J(6, -1)$, $K(10, -2)$, and $L(5, -3)$. Graph $\triangle JKL$ and its image after a translation along $\langle 0, 4\rangle$ and a reflection in the y-axis.

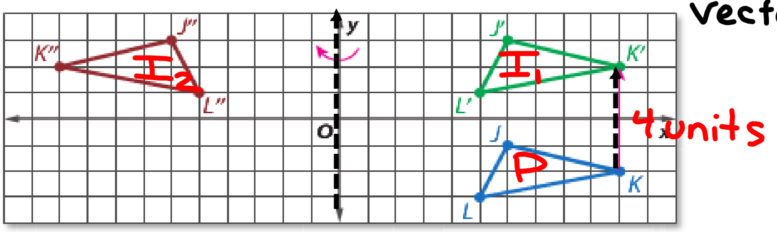
Step 1 translation along $\langle 0, 4\rangle$

$(x, y) \rightarrow (x, y + 4)$
$J(6, -1) \rightarrow J'(6, 3)$ (6,-1+4)
$K(10, -2) \rightarrow K'(10, 2)$ (10,-2+4)
$L(5, -3) \rightarrow L'(5, 1)$ (5,-3+4)

Step 2 reflection in the y-axis

$(x, y) \rightarrow (-x, y)$
$J'(6, 3) \rightarrow J''(-6, 3)$
$K'(10, 2) \rightarrow K''(-10, 2)$
$L'(5, 1) \rightarrow L''(-5, 1)$

Step 3 Graph $\triangle JKL$ and its image $\triangle J''K''L''$.



vector || y-axis
↑
is parallel to

Triangle PQR has vertices P(1, 1), Q(2, 5), and R(4, 2). Graph triangle PQR and its image after the indicated glide reflection.

1A. Translation: along $\langle -2, 0\rangle$ $(x-2, y)$
Reflection: in x-axis $(x, -y)$

$P(1,1) \rightarrow P'(1-2,1) = (-1,1)$

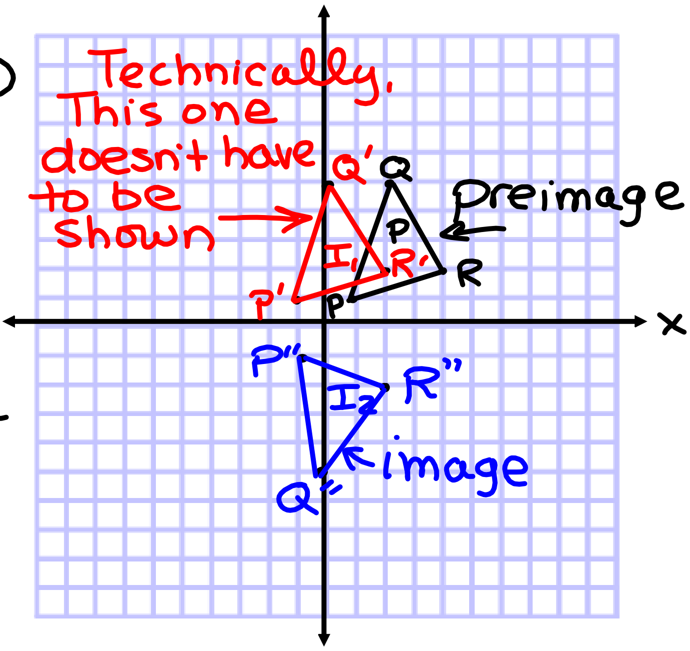
$Q(2,5) \rightarrow Q'(2-2,5) = (0,5)$

$R(4,2) \rightarrow R'(4-2,2) = (2,2)$

$P'(-1,1) \rightarrow P''(-1,-1)$

$Q'(0,5) \rightarrow Q''(0,-5)$

$R'(2,2) \rightarrow R''(2,-2)$



Triangle PQR has vertices P(1, 1), Q(2, 5), and R(4, 2). Graph triangle PQR and its image after the indicated glide reflection.

1B. Translation: along $\langle -3, -3 \rangle$ $(x-3, y-3)$
Reflection: in $y = x$ (y, x)

$P(1,1) \rightarrow P'(1-3, 1-3) = (-2,-2)$

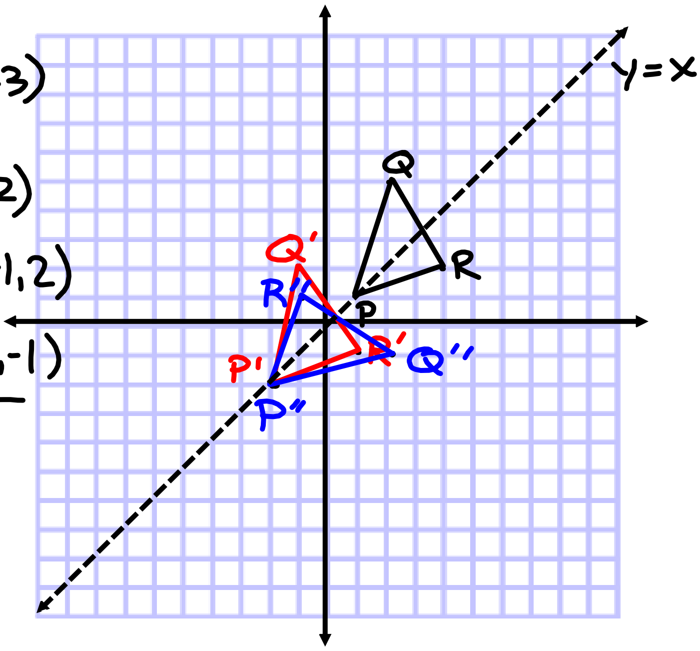
$Q(2,5) \rightarrow Q'(2-3, 5-3) = (-1,2)$

$R(4,2) \rightarrow R'(4-3, 2-3) = (1,-1)$

$P'(-2,-2) \rightarrow P''(-2,-2)$

$Q'(-1,2) \rightarrow Q''(2,-1)$

$R'(1,-1) \rightarrow R''(-1,1)$



In Example 1, $\triangle JKL \cong \triangle J'K'L'$ and $\triangle J'K'L' \cong \triangle J''K''L''$. By the Transitive Property of Congruence, $\triangle JKL \cong \triangle J''K''L''$. This suggests the following theorem.

Theorem 9.1 Composition of Isometries

The composition of two (or more) isometries is an isometry.

So, the composition of two or more isometries—reflections, translations, or rotations—results in an image that is congruent to its preimage.

AKA isometry AKA congruence transformation

StudyTip

Rigid Motions Glide reflections, reflections, translations, and rotations are the only four *rigid motions* or isometries in a plane.

ReadingMath

Double Primes Double primes are used to indicate that a vertex is the image of a second transformation.

Example 2 Graph Other Compositions of Isometries

The endpoints of $\overline{CD}$ are $C(-7, 1)$ and $D(-3, 2)$. Graph $\overline{CD}$ and its image after a reflection in the x-axis and a rotation 90° about the origin.

Step 1 reflection in the x-axis

$(x, y) \rightarrow (x, -y)$ ← rule

P $C(-7, 1) \rightarrow C'(-7, -1)$ I_1

$D(-3, 2) \rightarrow D'(-3, -2)$

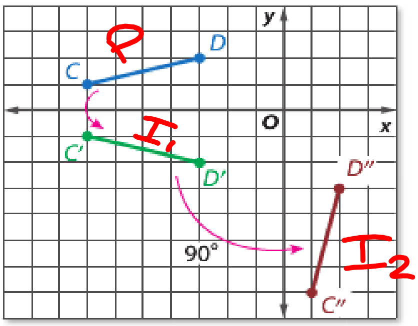
Step 2 rotation 90° about origin

$(x, y) \rightarrow (-y, x)$ ← rule

I_1 $C'(-7, -1) \rightarrow C''(1, -7)$ I_2

$D'(-3, -2) \rightarrow D''(2, -3)$

Step 3 Graph $\overline{CD}$ and its image $\overline{C''D''}$.



ReadingMath

Double Primes Double primes are used to indicate that a vertex is the image of a second transformation.

Triangle ABC has vertices $A(-6, -2)$, $B(-5, -5)$, and $C(-2, -1)$. Graph $\triangle ABC$ and its image after the composition of transformations in the order listed.

2A. Translation: along $\langle 3, -1 \rangle$
Reflection: in y-axis

$(x,y) \longrightarrow (x+3, y-1)$

$A(-6,-2) \rightarrow A'(-6+3, -2-1)$

$A'(-3,-3)$

$B(-5,-5) \rightarrow B'(-5+3, -5-1)$

$B'(-2,-6)$

$C(-2,-1) \rightarrow C'(-2+3, -1-1)$

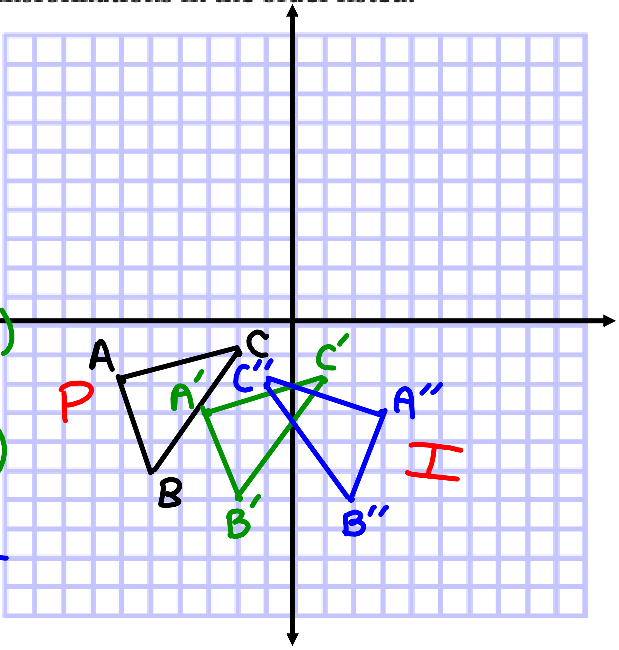
$C'(1,-2)$

$(x,y) \longrightarrow (-x, y)$

$A'(-3,-3) \rightarrow A''(3,-3)$

$B'(-2,-6) \rightarrow B''(2,-6)$

$C'(1,-2) \rightarrow C''(-1,-2)$



Triangle ABC has vertices $A(-6, -2)$, $B(-5, -5)$, and $C(-2, -1)$. Graph $\triangle ABC$ and its image after the composition of transformations in the order listed.

2B. Rotation: 180° about origin
Translation: along $\langle -2, 4 \rangle$

$(x,y) \to (-x,-y)$

$A(-6,-2) \to A'(6,2)$

$B(-5,-5) \to B'(5,5)$

$C(-2,-1) \to C'(2,1)$

$(x,y) \to (x-2, y+4)$

$A'(6,2) \to A''(6-2, 2+4)$

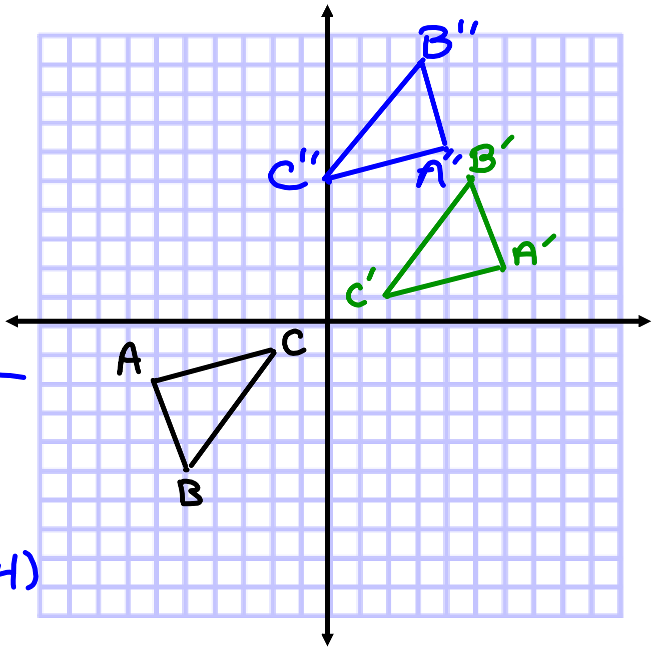
$A''(4,6)$

$B'(5,5) \to B''(5-2, 5+4)$

$B''(3,9)$

$C'(2,1) \to C''(2-2, 1+4)$

$C''(0,5)$

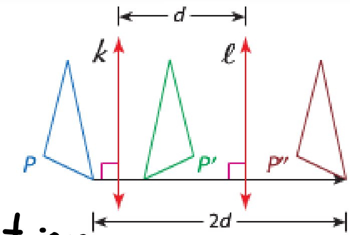


2 Compositions of Two Reflections The composition of two reflections in parallel lines is the same as a translation.

Theorem 9.2 Reflections in Parallel Lines

The composition of two reflections in parallel lines can be described by a translation vector that is

- perpendicular to the two lines, and
- twice the distance between the two lines.

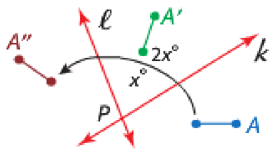


composition of 2 reflections over parallel lines = translation

Theorem 9.3 Reflections in Intersecting Lines

The composition of two reflections in intersecting lines can be described by a rotation

- about the point where the lines intersect and
- through an angle that is twice the measure of the acute or right angle formed by the lines.



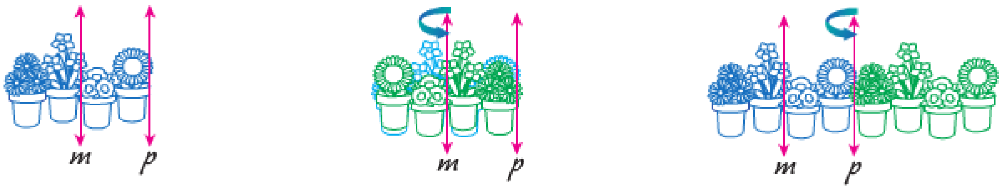
composition of 2 reflections over intersecting lines = rotation.

Real-World Example 4 Describe Transformations

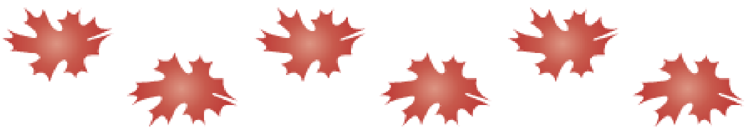
STATIONERY BORDERS **Describe the transformations that are combined to create each stationery border shown.**

a.

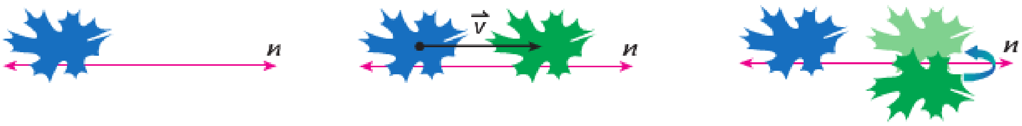
The pattern is created by successive translations of the first four potted plants. So this pattern can be created by combining two reflections in lines m and p as shown. Notice that line m goes through the center of the preimage.



b.

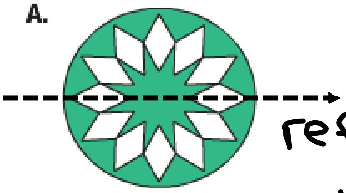


The pattern is created by glide reflection. So this pattern can be created by combining a translation along translation vector $\vec{v}$ followed by a reflection over horizontal line n as shown.



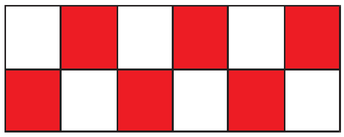
4. **CARPET PATTERNS** Describe the transformations that are combined to create each carpet pattern shown.

A.



reflection
+
rotation

B.



glide reflection

ConceptSummary Compositions of Translations		
Glide Reflection	**Translation**	**Rotation**
the composition of a reflection and a translation	the composition of two reflections in parallel lines	the composition of two reflections in intersecting lines